Toward Holistic Design of Spatial Packaging of Interconnected Systems With Physical Interactions (SPI2)

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Three-dimensional spatial packaging of interconnected systems with physical interactions (SPI2) design plays a vital role in the functionality, operation, energy usage, and life cycle of practically all engineered systems, from chips to ships. SPI2 design problems are highly nonlinear, involving tightly constrained component placement, governed by coupled physical phenomena (thermal, hydraulic, electromagnetic, etc.), and involve energy and material transfer through intricate geometric interconnects. While many aspects of engineering system design have advanced rapidly in the last few decades through breakthroughs in computational support, SPI2 design has largely resisted automation and in practice requires at least some human-executed design steps. SPI2 system reasoning and design decisions can quickly exceed human cognitive abilities at even moderate complexity levels, thwarting efforts to accelerate design cycles and tackle increasingly complex systems. Existing design methods treat pieces of the SPI2 problem separately without a fundamental systems approach, are sometimes inefficient to evaluate various possible designs, and present barriers to effective adoption in practice. This article explores a vision of a holistic SPI2 design approach needed to develop next-generation automated design methods capable of rapidly producing viable SPI2 design candidates. We review several technical domains related to holistic SPI2 design, discuss existing knowledge gaps and practical challenges, examine exciting opportunities at the intersection of multiple domains that can enable comprehensive exploration of SPI2 design spaces, and present one viable two-stage SPI2 design automation framework. Holistic SPI2 design opens up a new direction of high industrial and societal relevance for the design research community.

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1 Introduction

SPI2, which is a new term introduced in this article, stands for *Spatial Packaging of Interconnected Systems with Physical Interactions*, involves the spatial arrangement of components and interconnects inside irregular three-dimensional (3D) volumes as shown in Fig. 1 (e.g., the complex arrangement of components and connections underhood of a modern car, the performance of which is influenced by spatial thermal, fluidic, electromagnetic, and other phenomena). SPI2 design problems are extremely difficult to navigate. Holistic consideration of topological, geometric, and physics-based elements of SPI2 design is an unsolved problem of central importance to some of humanity's most pressing needs (e.g., energy, transportation, medical devices, microelectronics, and more). SPI2 design practice is making very incremental progress in archaic manual processes, and technological progress and society are suffering as a result. Current SPI2 design still relies largely on human intuition and manual spatial configuration, has resisted automation, and quickly exceeds human cognitive abilities at moderate levels of complexity. Many elements of engineering system design have advanced well beyond the low level of maturity that available SPI2 capabilities are currently stuck at. This lack of SPI2 design method maturity is currently a dominant bottleneck for a very broad range of engineering design efforts, resulting in (1) excessive system design time and resource requirements, (2) limited sophistication with which SPI2 problems are approached, and (3) the current reality that the packaging of many new technologies requires several years to evolve sufficiently for them to compete with efficiently packaged legacy technologies.

Transition to holistic SPI2 design is a crucial ingredient for moving many novel technologies, waiting in the wings as promising innovations, toward real systems, impacting areas such as fuel emissions [1], energy efficiency [2,3], and miniaturization of medical devices [4–10]. Although SPI2 problems play a central role in the functionality, operation, energy usage, and life cycle of practically most engineered systems, the minimal level of support provided by the currently available design automation tools to the design of SPI2 systems hinders their progress. Consequently, new SPI2 design automation methods are needed that can further reduce the size of complex systems considerably, impacting applications such as power-dense smart batteries [1], spacecraft cooling systems [11], satellites [12,13], minimally invasive medical wearables [8–10], vehicles with more usable volume [14], and compact avionics and military electronic systems [15,16]. Engineers have labored for decades to improve spatial packaging density across diverse domains such as avionics [17–20], spacecraft systems [21], automotive packaging [22], vehicle electrification [23], and spacesuit design [24,25]. These advances, however, have largely been incremental and have depended heavily on the ingenuity of human experts. The more sweeping advances needed in SPI2 design and sophistication are hindered by the current lack of a unified SPI2

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Interconnected components within a packaging volume

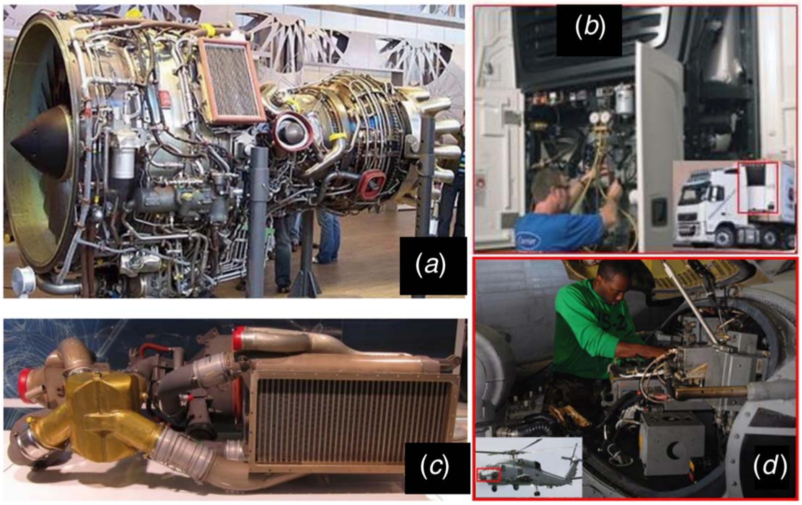

Fig. 1 Diverse examples of systems that present 3D SPI2 spatial packaging and routing complexity, subject to physical interactions, and exhibiting spatial accessibility challenges for life-cycle processes (all of which typically involve manual design): (*a*) the externals (components, wires, pipes, and ducts interconnecting components and engine features) of a commercial turbofan engine covering the limited surface area of its core and fan case, (*b*) the refrigeration unit for a truck trailer, (*c*) an environmental control system providing pressurization and cooling to commercial aircraft cabin air, and (*d*) helicopter avionics hardware, interconnected by wire harnesses and thermal management pipes and ducts to reject electronics heat, presenting accessibility challenges in the front avionics bay

design theory and associated practical methods. Other engineering design domains, such as material distribution topology optimization (MDTO) [26] and aeroservoelastic system design [27], have realized rapid progress in design capability and societal impact by successfully leveraging powerful design automation methods to help navigate design spaces that are too complex for expert human cognition alone. Advancement in SPI2 system design will require similar formalisms and methods that do not yet exist. SPI2 design has been resistant to automation, in part, due to a lack of appropriate design representations for comprehensive SPI2 problems that are compatible with potential design automation strategies.

Not all real-world interconnected systems require a holistic SPI2 design approach. For instance, systems such as landline phones, electric clothing irons, and computer mice also contain components, interconnects, and some physics-based design considerations. However, these packaging design problems can be solved manually with existing tools. They are neither large in scale nor have tight coupling between geometry and multiphysics domains. Thus, they can be designed successfully utilizing existing tools and human design ingenuity. However, holistic SPI2 design is essential for accelerating the advancement of systems such as aircraft, satellites, automotive engines, biomedical devices, and emerging technologies that have intricate geometries, complex topological properties, multiphysics interactions, complex manufacturability and operational requirements, and system dynamics considerations. Hence, making even merely satisfactory SPI2 design decisions is quite challenging where one must account for the inherent coupling that exists between different domains and design processes.

1.1 Objectives.

The primary goal of this article is to define the holistic SPI2 design problem, review its constituent research fields, identify existing technical gaps and challenges, provide a vision for design research teams to address these gaps in SPI2 design theory and capability, and catalyze the creation of powerful new SPI2 design methods and knowledge to take full advantage of the rich and complex design spaces associated with SPI2 systems. This will enable practicing engineers to go beyond what is possible using existing methods (usually based on packaging and routing design rules [28,29], design heritage [30–32], and expert intuition [33,34]) and (1) mitigate the costly packaging bottleneck in 3D system design, (2) enable a step change in the complexity of systems that can be optimally packaged, and (3) produce greater system performance and functionality, with much smaller footprints, by explicit treatment of complex design couplings through integrated design optimization methods. It must be noted that this article serves as a preliminary attempt to consolidate different related aspects of the 3D SPI2 problem. The authors believe that there is a significant opportunity for broader advances in SPI2 design knowledge leading to powerful SPI2 methods and tools, enabling a wide range of better-engineered systems.

The remainder of this article is organized as follows. In Sec. 2, we define the holistic 3D SPI2 design problem and its attributes. The differences between 3D SPI2 design and 2D very large scale integration (VLSI) design are also discussed in detail. The individual elements of SPI2 design research are discussed, and previous related work in these areas is reviewed in Sec. 3. In Sec. 4, critical gaps in SPI2-related research are articulated. Section 4.1 outlines the challenges associated with integrating the different SPI2 problem elements. In Sec. 5, a vision for SPI2 research with the potential for significant impact is introduced. Section 6 discusses one viable SPI2 design framework, which is a two-stage approach that was recently developed by the authors who utilize new mathematical representations to tackle SPI2 design problems holistically. Finally, concluding remarks are presented in Sec. 7.

2 Spatial Packaging of Interconnected Systems With Physical Interactions Problem Definition and Its Key Attributes

The 3D Spatial Packaging of Interconnected Systems with Physical Interactions (3D-SPI2) problem can be defined as the optimal spatial arrangement of heterogeneous geometric components and interconnects of often non-negligible sizes inside irregular three-dimensional volumes, along with the consideration of their physics-based behavior, life-cycle processes, and system operating conditions. These design problems cut across a wide swath of engineered-system domains that are vital to society (e.g., medical devices, transportation, and computing hardware), and entail especially large design spaces (combining complex combinatorial/topological, geometric, parametric, and time-dependent decisions) that are difficult to navigate via either expert human cognition or computational search. These have resisted holistic treatment by potentially powerful design automation methods and still rely largely on manual (and sub-optimal) spatial placement by designers supported by computer-aided design (CAD) tools. Designing SPI2 systems still require highly skilled engineers who understand the engineering operation, manufacturing, assembly, testing, maintenance, and repair requirements. Moreover, design and maintenance of large-scale systems such as aircraft and ships require thousands of person hours. During maintenance and repair, these systems' capabilities are unavailable, thus increasing the required sizes of fleets and the associated cost. Any advancement to overcome this bottleneck has the potential for significant technical and economic impact.

The SPI2 design problem:

(1) Is fundamentally three-dimensional and involves interconnected heterogeneous physical subsystems and components with complex geometry and topology that functions within often complex, irregularly shaped enclosing volumes?
(2) Includes interconnects of various types (ducts, pipes and/or wires, etc.), sizes, shapes, and requirements (curvature, proximity, non-proximity (if components need to be far apart), temperature, electromagnetic interference (EMI), etc.) as well as various levels of spatial and topological complexity, as illustrated in Fig. 1.
(3) Is governed by strongly coupled physical interactions (e.g., thermal, hydraulic pressure, electromagnetic, and thermomechanical) and by the influence of the spatial arrangement on system behavior and performance?
(4) Is characterized in 3D by solid components (casings, bays, etc.) that may have holes or spatial-access ports? This, in turn, makes the topological considerations more subtle and complex.[1] For example, an interconnect may pass through a hole of a component or bypass the hole and be routed between components.
(5) Must meet a diverse set of constraints that depend on a variety of functionally related considerations, such as geometry (for ensuring both feasibility and connectivity), physics, material behavior, failure mechanisms, assembly/disassembly, component accessibility, manufacturing, and repair.
(6) Must attain desired value metrics: spatial packaging density, volumetric power density, product life-cycle costs, system efficiency, and system reliability.

2.1 Complexity of Spatial Packaging of Interconnected Systems versus VLSI. Significant work has been performed in VLSI circuit component layout, design, and routing optimization [35,36] for several board-based electronic applications. To an outside observer, it may come as a surprise that the VLSI design problem has been automated successfully while mechanical design automation continues to be an active area of research. However, the reasons are not merely academic and the fundamental differences between the two classes of design problems have been carefully documented some time ago [37]. On the one hand, VLSI circuits are complex and efficient 2D systems [38] whose design is carried out by a few well-integrated design tools. Moreover, the same manufacturing process can be used to fabricate practically any VLSI circuit that can be designed. On the other hand, mechanical designs, and specifically real-world SPI2 designs, are 3D systems (which adds a significant layer of complexity) and have multiple diverse attributes such as components with complex spatial geometries (including concave and convex surfaces), restrictive domains, arbitrarily sized, irregularly shaped bounding volumes, interconnects of various types (pipes, ducts, and/or wires, etc.) and radii, possible topological network configurations [39,40], strongly coupled physical interactions (thermal, hydraulic pressure, electromagnetic, etc.), are often large scale and frequently encompass several other design challenges. A large number of corresponding design tools are not well integrated; the manufacturing processes are highly specialized, and human judgment, even using available software tools, is insufficient to attain accurate, optimal designs, or compare their size, weight, performance, and cost. In addition, these systems need to meet spatial accessibility constraints to support safe and efficient manufacture, assembly, maintenance, diagnosis, overhaul, repair, upgrade, replacement, and complex operational requirements.

3 Spatial Packaging of Interconnected Systems With Physical Interactions Problem Elements

The SPI2 design problem consists of different intricately related technical elements that are individually very challenging themselves. Practically most interconnected engineered systems contain a set of similar or heterogeneous components and subsystems that comprise the system, the desired interconnects, the physical environment in which the system operates, the functional constraints, the operating spatial envelope, and the manufacturability, assembly/disassembly constraints. The *SPI2 design goal* is to find the optimal 3D spatial component placement and routing of the interconnects that meet a variety of constraint types (geometric, topological, physical, structural, manufacturing, etc.) and maximize system value. As illustrated in Fig. 2, solving the SPI2 design problem requires methods to solve (1) the 3D component placement and layout design [41,42] as well as (2) the 3D interconnect routing in a (3) physics-based optimization framework, by utilizing (4) appropriate SPI2 design abstractions and representations that adequately support the required geometric-, functional (connectivity, modularity)-, topological-, and optimization-related computations. While additional considerations may interact with the SPI2 system design (e.g., detailed component design, control systems, systems of systems factors), these four elements of research are the focus of this article as they play a central role in developing holistic SPI2 design methods. We review in the following subsections the state-of-the-art methods in each of these above four areas as they relate to SPI2 system design.

There exists some prior work where some of these above SPI2 elements have been combined, such as integrated component placement and routing design methods in Refs. [43–46]. An earlier survey by Cagan et al. [46] discusses some optimal component placement and routing methods that only focused on the spatial and geometric design problems. For instance, in Ref. [44], the authors combined multiple objective functions, the component packaging density, the total routing length, and overall system volume objective functions into a weighted-sum objective function. The interference detection constraints were added as penalty terms to this objective function to avoid infeasible solutions. Previous optimization methods do address some SPI2 interaction elements but did not fully account for the inherent coupling that exists between geometric, topological, functional, operational, and multiphysics interactions (e.g., thermal properties, EMI, and hydraulics) between components, interconnects, and the surrounding environment. Furthermore, the design framework and computational models for these previously developed methods did not illustrate the addition of other vital design considerations such as life-cycle costs for maintainability, ease of monitoring and diagnosis, accessibility, assembly and disassembly, repair, and so on. The existing

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[1] Compare, for example, the set theoretic complement of a 2D solid with holes, with the complement of the 3D solid obtained by extruding the same 2D profile. The former is a 2D disconnected set, while the latter is a 3D connected set.

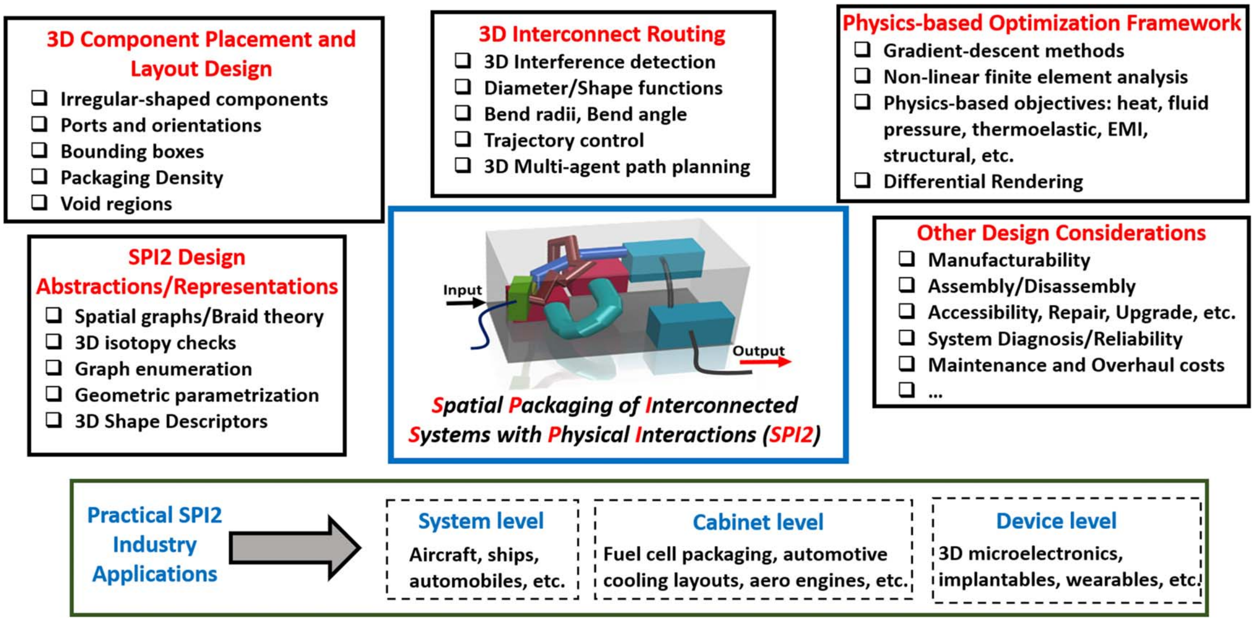

Fig. 2 SPI2 system design research has multiple facets, including four identified key design problem elements (3D component placement and layout design, 3D interconnect routing, physics-based optimization framework, design abstractions/representations), and other design considerations such as lifecycle and operational process metrics. Furthermore, SPI2 design research is applicable to a wide variety of practical industry-relevant problems at various design scales (system-, cabinet-, or device-level applications).

methods may perform much better due to the availability of powerful computational resources now than two decades ago. These methods did not illustrate holistic SPI2 design optimization due to missing integration between specific SPI2 problem elements. Hence, there is a need to carefully review existing capabilities, explore the gaps, and efficiently integrate the different SPI2 design features to advance the field. This review article does not suggest neglecting prior work in spatial packaging but aims to integrate promising techniques with new, flexible, computationally efficient, and robust SPI2 design automation methods.

3.1 Three-Dimensional Component Placement and Layout Design. Three-dimensional component layout design is a 3D bin-packing problem [41] that can be formulated as a mathematical optimization problem involving an optimal placement and orientation of components or objects [47–49] within a given 3D volume based on some appropriately defined objective function and constraints [50,51]. Unfortunately, these problems are known to be NP-Hard in combinatorial optimization [44,52]. An NP-Hard problem is not solvable in polynomial time but can be verified in polynomial time. Typical engineering systems are a combination of functionally and geometrically interrelated components. The spatial location and orientation of these components affect several physical quantities of interest to the designer, engineer, manufacturer, and the end user of the product. The 3D component layout design concerns itself with determining the optimal spatial location and orientation of a set of components given some design objectives and constraints. A typical 3D component placement formulation models the layout problem as a volume minimization problem, often combined with a weighted sum of other design objectives and penalties for constraint violation. Design objectives can include a variety of relevant metrics such as the amount of cable used in the engine compartment of a car, the power density of an electronic component, the packaging density of a drill, or the center of mass of a space vehicle. A key constraint is the non-intersection of components [53] and non-protrusion of components outside the design space. Other constraints include spatial relationships between components (e.g., turbomachinery co-axially mounted on a shaft) and between a component and the packaging volume (e.g., gravity-based orientation of fluid reservoirs).

The 3D geometric packaging problem is often formulated as an optimal component placement layout problem [54,55], where component geometries can be arbitrary [56–58], with multiple types of design goals and spatial constraint satisfactions [46]. For practical purposes, the minimization of layout cost functions is done under certain constraints imposed by design, fabrication, and operational requirements. Most layout algorithms are restricted to a certain class of systems, and problem scale is limited (solution can quickly become intractable due to its combinatorial nature). Problem variants differ by the particular definition of their packaging constraints (presence of guillotine cuts, balancing and stability of the packaging, possible overlapping of certain items, forbidden rotations of the items, etc.) and objective function, going by the well-known names of knapsack, bin packing, strip packing, variable-sized pellet packing, container loading, etc. Design automation methods for solving the optimal spatial packing problem have been developed and studied previously in the context of many applications, such as vehicle assembly [59], electronic module layout design [45,60,61], 3D container loading [62], bin packing [63], computer animation [64], the layout of components in additive manufacturing [65], and automotive transmission design [66]. Solution algorithms used in previous 3D layout design research can be generally classified under three categories: gradient-free algorithms [67,68], heuristic methods [30,32], and gradient-based algorithms [44,69–71]. Optimization approaches have incorporated metrics such as packaging volume and mass properties [72,73], and additional solution methods include pattern search [74,75] and ant colony optimization [76].

Finally, recent progress in the field of 3D robotic manipulation has generated interest in fully automatic robot bin picking and dense object packing in warehouses [77–79]. This can be leveraged for the automatic assembly of SPI2 systems. The 2017 Amazon Robotic Challenge (ARC) required stowing items into a storage system, picking specific items, and packing them into 3D boxes. This approach requires tight coupling between several disciplines such as computer vision, motion planning, robot grasp planning, and control. Picking random objects of different sizes and shapes within cluttered environments is a very challenging task. This work is highly related to SPI2 and can therefore be leveraged for SPI2 design automation using robots, except that interconnect routing and physics aspects are also involved. Similarly, robotics research in tactile sensing for manipulating cables, wires, and other deformable objects [80] can be leveraged for performing complex interconnect routing in tightly constrained industrial applications. Both robot object picking and robot cable manipulation can be integrated together in future to design hybrid robots that can assist practicing SPI2 designers and engineers.

3.2 Three-Dimensional Interconnect Routing. Since the 1970s, 3D interconnect/pipe routing design has been studied in various industrial domains [81], such as transportation [82,83], chemical process plants [84], oil and gas refineries [85], water treatment and distribution [86], hydroelectric power [87], robotic path planning [88,89], large-scale integrated circuits [90,91], and computing hardware [92,93], and represents one of the most important aspects of systems and operation integration. However, due to the complexity of routing systems and the diversity of constraints involved, it is quite time-consuming and difficult to achieve a feasible routing design using both manual experience and CAD-based design tools.

Historically, 3D pipe layout has been approached as a two-stage process. The first stage is optimal equipment allocation, i.e., finding the spatial location of the equipment to minimize some measure of cost and satisfy existing constraints, such as maximum distances between components and maintenance access requirements. For routing based on a regular grid, Manhattan distances are often used to roughly evaluate the total cost. The second stage is the determination of the 3D pipe routes that avoid collisions and meet other constraints. Of course, the two steps are not independent of each other. As a result, the sequential execution of these two stages must be iterated to identify an acceptable design. Several options for pipe layout solution algorithms exist. Route planning algorithms have been developed for the last six decades, starting with the well-known Dijkstra's algorithm [94] for computing the shortest path on a graph between two nodes. Various extensions and modifications have been proposed to improve the search efficiency, including the heuristic algorithm proposed in Ref. [95] and those presented in Refs. [96] and [97]. In 1961, Lee [98] proposed a maze algorithm to solve the problem of connecting two points. Since route planning can be formulated as an optimization problem [99], there have been many instances of modern optimization algorithms applied to optimal path planning, including those based on genetic algorithms [68,100,101], ant colony [76,102–104], and particle swarm optimization [28,105], simulated annealing [44,71], pattern search [74,75], and several other heuristic methods [32,106].

Moreover, CAD-based routing algorithms [29,107,108] and cable simulation algorithms [109,110] using additive manufacturing technology have also been proposed for 3D pipe routing. With the advent of autonomous robots, Dijkstra's algorithm has also been extended to dynamic domains. Importantly, most of these routing algorithms do not consider physical phenomena other than perhaps particle dynamics.

Several related papers have been published across multiple engineering disciplines, including electrical, chemical, and aerospace engineering. 2D routing algorithms were developed for VLSI circuits with fixed layouts based on the Manhattan distance and its variants [90]. Other 3D routing applications include aero-engine externals routing [111,112], ship pipe routing [108,113,114], electrical harnesses routing for vehicles [115], chemical plant pipe routing [84], electrical wire routing in buildings [116,117], field-programmable gate array design [118], piping for airbus landing gear bay [119], unmanned aerial vehicle navigation [88], vehicle routing [106], and robotic path planning [89,120].

Finally, it is interesting to note that the 3D pipe routing problem, which aims at placing non-intersecting pipes between different components has been solved using the multi-agent path finding (MAPF) algorithms [121] in robotics research as discussed in Refs. [122–124], where robots (or agents) should move to given target locations from start locations in a known 3D system environment. 3D MAPF research could thus provide a basis for addressing the 3D interconnect routing problem, noting that the former is dependent on the dynamics of agents and the latter typically is a static problem. These Refs. [122–124] also appeal to the community for a deeper exploration and implementation of the MAPF algorithms for practical and complex 3D interconnect pipe routing problems. We have identified two promising MAPF algorithms that can be appropriately modified and integrated into a holistic SPI2 design automation framework. They are (1) Prioritized Planning (PP) algorithms [125–127], wherein the pipes can be ordered sequentially according to some fixed priority. Each higher-priority pipe then becomes an obstacle for all lower-priority pipes that fall in the sequence; and (2) Priority-Based Search (PBS) [128], wherein it allows several pipes to be routed simultaneously.

Commercial software tools such as Siemens—Flexible Pipe, SolidWorks Routing, M4 plant, Bentley AutoPIPE, AutoCAD P&ID, COMSOL Pipe Flow, ProCAD P&ID, MATPIPE, CAD-profi HVAC & Piping, and others focus only on routing and piping design for fixed component layouts. An exhaustive list of CAD piping and plant design tools can be obtained from the resources in Ref. [129]. In summary, these tools are very powerful piping design tools but cannot be directly used for holistic spatial packaging design as they are not integrated with component placement, system operational management challenges such as component diagnosis, replacement and repair, and physics-based system-level performance evaluations.

3.3 Physics-Based Topology Optimization. As mentioned earlier, an important aspect of the SPI2 design research is to integrate the physical interactions between the various components, interconnect flow passages, and any other elements as part of the spatial component placement and routing optimization problem. Topology optimization, defined here as the optimal placement of material in a 2D or 3D geometric domain, does take into account models of physical behavior (e.g., thermal, fluid, electromagnetic, and mechanical stress). This method class has been used across a range of engineering domains, including structural design for maximum stiffness [130], multi-material properties [131], or component geometries for optimal heat conduction properties [132,133]. Problems that include multiple physical phenomena have also been considered. De Kruijf et al., Takezawa et al. and Kang and James performed optimization studies which included both structural and thermal conduction requirements [134–137]. The aerodynamic shape and internal structure of a wing have been optimized simultaneously [138–140], considering the interaction between aerodynamic loading and structural wing response. Topology optimization has also been used to optimize the placement of components and their supporting structure [141,142]. Several feature-mapping methods [143], including using B-spline curves [144–146] and level-set methods [147–149], exist that can be leveraged to perform multiphysics multicomponent topology optimization. This allows sections of specific geometry, such as a pattern of bolt holes, to be distributed optimally within a structure. Designs produced by topology optimization are often infeasible for traditional manufacturing methods (subtractive, formative), but often can be made using additive manufacturing [150]. The design of components that are more easily manufactured using traditional methods motivates the development of methods that optimize designs made from standard material sizes and shapes, typically using ground structure methods [31,151]. The geometric projection methods in Refs. [152,153] have also been suggested to optimize structures made from stock materials.

Recent developments made in the geometric projection method [152,154] and integrated layout design of multicomponent systems [141,155–158] using topology optimization are highly relevant to SPI2 design research because these methods allow the

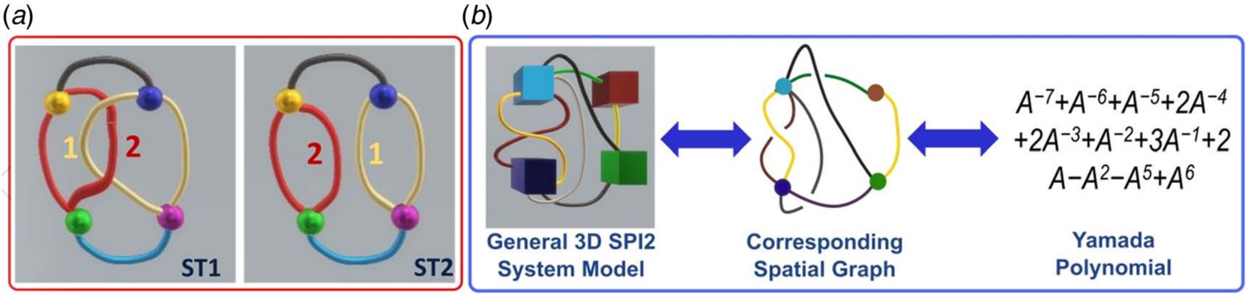

Fig. 3 (*a*) Spatial topologies one (ST1) and two (ST2) have the same system connectivity, but are two unique 3D spatial topologies (STs) as there is *no continuous deformation of component locations and interconnect trajectories* that can morph one topology into the other. Observe that in ST1, interconnects 1 and 2 are entangled (or linked) together while in ST2, they are free (interconnects 1 and 2 cannot be continuously deformed in one topology to attain the other topology) and (*b*) spatial graph representation of a 3D SPI2 system and its corresponding Yamada polynomial (a mathematical abstraction that can be operated upon).

decoupling of geometric parameterization from solution strategy. An initial investigation by the authors of using geometric projection methods for 2D SPI2 design problems can be found in Refs. [159–162]. The simultaneous physics-based packaging and routing approach utilized in Refs. [163,164] makes significant system volume reduction possible for spatial packaging. The projection method of Norato et al. [152] was extended to allow devices of arbitrary polygonal shape to be projected. Sensitivity analysis for this projection is provided to allow the efficient use of gradient-based optimization methods. These methods could be extended to model various combinations of physics; for example, fluid-thermal, thermal-electric or structural-fluid systems. Finally, the geometric projection method is essentially spatial occupancy and derivatives, which has been used in various contexts for a long time. It is one of several ways to connect an explicit parameterization to a grid in a way that supports automatic differentiation.

In summary, standard topology optimization deals with material distribution within homogeneous design spaces, while SPI2 involves spatial placement and connectivity of heterogeneous components in heterogeneous design spaces characterized by coupled physical phenomena. This implies that the corresponding optimization methods cannot be interchangeably used.

3.4 Mathematical Abstractions and Representations for Spatial Packaging of Interconnected Systems With Physical Interactions Design Problems.

A vital aspect of any engineering design optimization problem is the choice of mathematical abstraction and design representation used for system modeling. On the one hand, mathematical abstractions must capture the system attributes that are relevant to design decisions. Moreover, the design representations must have the accuracy and compatibility that are needed to support the required computations in an efficient manner. For example, the abstractions used for SPI2 problems must capture the geometry and topology of the SPI2 systems, and the SPI2 representations must integrate with physics considerations, detailed geometric analysis, as well as navigation of formidable spatial topology (ST) decision spaces as shown in Fig. 1(*a*). Since one of the main objectives of the SPI2 design problem is to determine the optimal spatial location and topology of the interconnects (such as ducts, pipes, or wires), in the space defined by the spatial envelope as well as the system components and subsystems, these decisions represent some of the most difficult elements of SPI2 problems. Unlike 2D systems, 3D systems may contain crossings (i.e., junctions where two interconnects go over or under each other), and it is important to have design abstractions and representations that can adequately capture the geometry and topology of the space and adequately support the search for optimal solutions. The energy transfer through SPI2 system elements materializes through complex geometric interfaces of geometric models that may exist in different geometric representations. Importantly, no commercial CAD system or, to the best of our knowledge, even research systems can handle models that have such a representation mismatch and allow their geometry and spatial topology to simultaneously evolve as the optimization progresses. Therefore, it is essential to develop unified geometric representations (UGRs) such as Maximal Disjoint Ball Displacement (MDBD) method [165] to create novel models for quantifying the geometric interfaceability of geometric models to support highly coupled packaging optimization.

It must be noted that, even without considering physical aspects, the 3D spatial packaging problem is exceptionally difficult. In solving complex design optimization problems, much depends on the mathematical abstractions and representations that are used to describe the various features of this system and system classes. To date, the authors have identified three important mathematical abstractions/representations that are suitable for enumeration (individually listing or counting the possible number of designs) and optimization of different initial SPI2 topological designs, namely spatial graphs, braids, and homotopy classes (as shown in Figs. 3 and 4. A description of these three abstractions follows:

(1) *Spatial Graphs* are graphs embedded in a metric space [166]. A metric space is a non-empty set together with a metric on the set. The metric is a function that defines a concept of distance between any two members of the set, which are usually called points. [167]. In our prior work [168], we have shown that spatial graphs can be used to represent, enumerate, identify, and generate unique 3D spatial configurations for 3D engineering system networks. An illustrative example is shown in Fig. 5. A task for future work is to demonstrate the usefulness of this method in industry practice. These graphs can take advantage of the theoretical and computational machinery developed in the field of graph theory, and their topological properties can be studied with tools developed in knot theory. Mathematically, knots are tame embeddings. An embedding is a representation of a topological object, manifold, graph, or other similar entity in a certain low dimensional space that preserves its connectivity or algebraic properties [169]. A tame embedding is a closed polygonal path in a three-dimensional space of circles in $R3$, and informally, knot theory provides the mechanisms needed to investigate whether a closed loop of strings, such as those appearing in spatial graphs, is knotted and whether we can deform the loop in question into a circle without cutting or breaking it. The spatial topology of a 3D system can be captured by a spatial graph where components are the nodes, interconnections are the edges, and the ports are node valencies. Node valency or node degree is the number of edges connecting it. For example, if the node valency is 2, then the node has two edges connected to it.

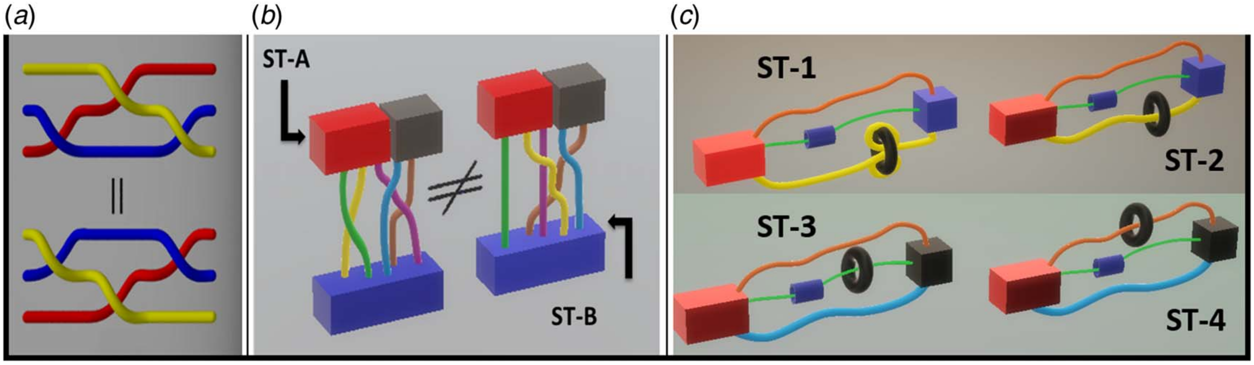

Fig. 4 (*a*) Two equivalent braids with corresponding braid words: [2 1 2] and [1 2 1], respectively, (*b*) two different 3D interconnected spatial topologies (STs) enumerated using braid-based enumeration, and (c) four different STs of a system containing both hollow and solid components. ST-1, ST-2, and ST-3 come under different homotopy classes as the interconnects cannot be continuously morphed through the hollow objects that they are passing through to attain the other 3D spatial topologies. ST-1 and ST-2 are different spatial topologies due to the knotted yellow interconnect.

(2) *Braids* are related to knots but differ from them in that they are made of strings that are not closed loops as shown in Fig. 4. The two concepts are related to each other via two well-known results due to Alexander and Markov. Although one can, in principle, use braid theory to study knot theory (and vice versa), there are reasons why one may need to use one or the other to answer specific topological questions. The details of these reasons are beyond the scope of this article. Importantly, braids and braid theory [170] can be used to abstract the interconnect network within a 3D SPI2 system, which allows an enumeration of various braid- based representations of the interconnect network, thus supporting the exploration of discrete topological system configurations. In this context, specific braid and knot equivalence methods can be leveraged to weed out redundant topologies. We observe that braid and knot theories have been successfully used in other applications such as protein folding [171] and, very recently, in multi-agent motion planning [172,173].

(3) *Homotopy classes*: Two continuous functions from one topological space are called homotopic if one can be "continuously deformed" into the other; such a deformation is called a homotopy between the two functions [174] and captures the notion of topological invariance of maps during continuous deformations. For example, in robot path planning, two paths with common and fixed endpoints are called homotopic if one path can be continuously deformed into the other without crossing the obstacles within the region. A homotopy class of paths is a collection of homotopic paths. Homotopy classes collect maps that share specific topological properties. Classification of homotopy classes in two-dimensional spaces has been studied in the robotics literature using geometric methods [175], probabilistic road-map construction techniques [176], and triangulation-based path planning [177]. There are many applications in robot motion planning [178] where it is important to consider and distinguish between different homotopy classes of trajectories (paths followed by robots). A strategy for classifying and representing homotopy classes in a three-dimensional configuration space, using theorems from electromagnetism, has been proposed [178]. BiotSavart's and Ampere's Laws were used to define a differential 1-form, the integration of which along trajectories gives an invariant for the homotopy classes of trajectories. This concept of homotopy classes has been extended to defining different classes of 3D SPI2 problems into two categories: (1) systems containing only solid components (closing infinite or unbounded objects), and (2) systems containing both solid and hollow components (decomposing objects with genus > 1). For example, Fig. 4(*c*) shows two different spatial topologies of a SPI2 system, where an interconnect in ST-3 passes through the torus, and outside the torus in ST-4, respectively. The paths taken by this interconnect in ST-3 and ST-4 are not homotopic (or in other words fall under two unique homotopy classes) as one path cannot be continuously deformed to the other without cutting the torus. Such mathematical extensions are very valuable in identifying unique system topologies for improving system design richness and flexibility.

4 Existing Gaps and Associated Challenges

The main limitation of the current approaches used in component placement, interconnect routing, and physics-based topology optimization is that they address these problems separately, rather than in a combined manner that accounts for inherent coupling. In addition, current methods being used in practice require frequent interactions with expert human designers and consider only geometric aspects of the problem while neglecting important physical system properties such as operating temperature/thermal loading, pressure drop, and aerodynamic and electromagnetic effects. Thus, existing methods may not extend well to the general coupled SPI2 design problem. Existing methods can be utilized but must be properly integrated into a new SPI2 design automation framework.

In addition, the amount of time required for a human designer to generate a feasible design and analyze its performance limits the ability of engineers to explore these complex design spaces within a constrained project timeline. Given sufficient time, existing strategies can produce some feasible designs, but they may not be optimal given all of the system requirements and design couplings, and the complexity of systems that can be considered is limited. In current practice, many aspects of the layout and routing problems are solved manually, which severely limits design capabilities for systems involving complex component layout design and routing tasks (especially in cases with strong physical interactions). In addition, the performance evaluation of the designs obtained from existing systems is left to human designers.

This article relies on the premises that (1) a holistic approach to the SPI2 design problem is needed to produce designs that are more compact, more complex, and higher-performing than current SPI2 systems, and (2) such an approach requires fundamentally different abstractions and design representations than those being used today.

We summarize here some of the most important knowledge gaps related to SPI2 design:

(1) *Lack of methods for systematic exploration of the SPI2 design space*: A critical gap is the lack of methods to comprehensively search a SPI2 design space, such as those that have recently become available for system architecture

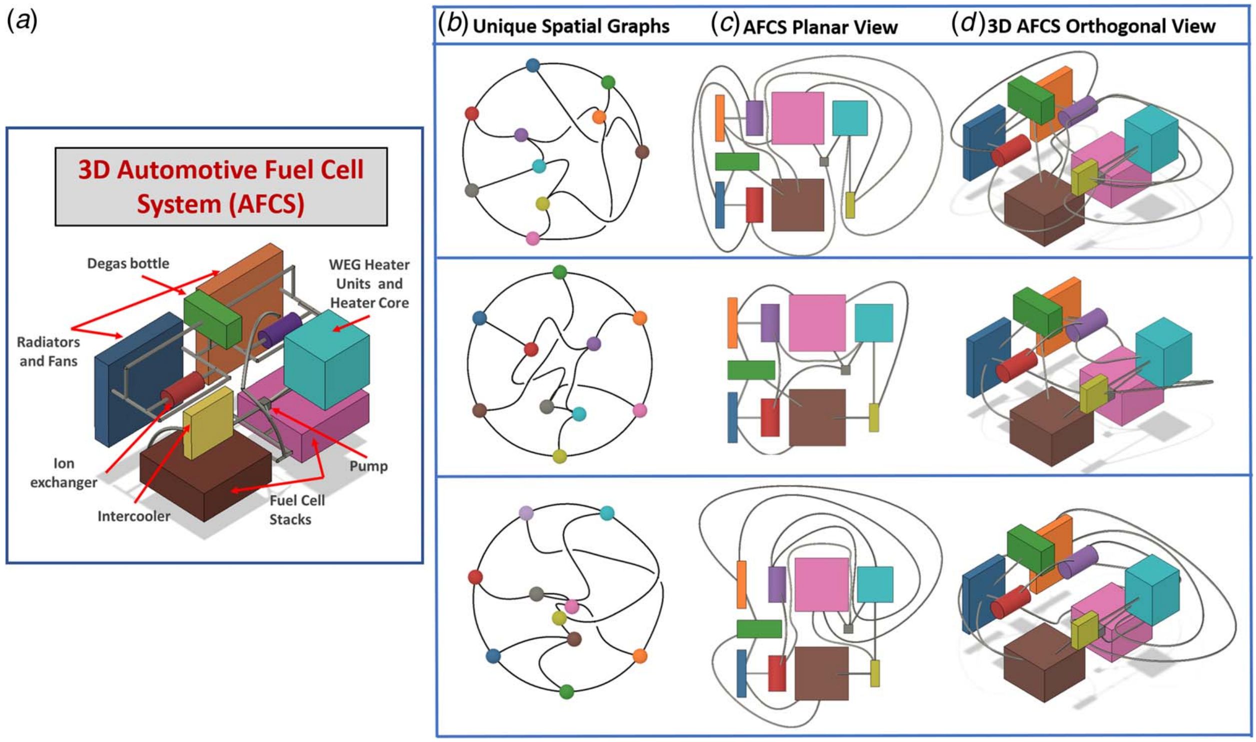

Fig. 5 This is an illustrative example of how spatial graph representations are used to enumerate unique spatial topologies (or configurations) of an automotive fuel cell system (AFCS); (*a*) shows the different components of the 3D AFCS and their interconnections, (*b*) shows a sampling of three of the several unique spatial graphs enumerated for the given AFCS, (*c*) and (*d*) are the planar and orthogonal views, respectively, of the 3D models generated from their corresponding spatial graphs shown in (*b*). The automated spatial topology enumeration method utilized here for navigating through the discrete 3D spatial topology options is discussed in our prior work in Refs. [161,168]. The different 3D models generated using spatial graphs can be utilized as initial design layouts for continuous multiphysics optimization in Stage 2 of the two-stage SPI2 design automation approach discussed in Sec. 6 of this article.

enumeration [179,180]. An efficient enumeration technique of the candidate solutions is required to navigate through the discrete 3D topology options possible for SPI2 design.

(2) *Need for handling continuous and discrete elements together*: One complicating attribute of the SPI2 problem is that it contains both continuous (spatial locations, interconnect diameter, trajectory, etc.) and fundamentally discrete (topology options, number of components, interconnects, crossings, etc.) elements. Such a mix of elements is very challenging for current optimization solvers. New design optimization techniques tailored to the SPI2 design problem that can efficiently navigate the hybrid design space are needed, such as the two-stage approach discussed in Sec. 6, and Ref. [160]. In addition, a unified geometric parameterization of both discrete and continuous variables would enhance the efficiency of the optimization process and aid with improved problem formulations.

(3) *Lack of common design language*: SPI2 design research exists along the interfaces of several engineering domains and applications. To communicate design knowledge effectively between various communities of practitioners and domain experts, there is a need for common terminology and constructs to address problem elements.

(4) *Need for flexible design representations*: Existing design representations are developed to support specific SPI2-related problem elements, such as component placement and routing treated independently, but cannot be utilized for creating general design methods for holistic SPI2 applications. Therefore, there is a need to develop more unified abstractions and representations that support holistic modeling and that can capture the various SPI2 problem features with sufficient expressivity.

(5) *Tailored SPI2 routing algorithms*: Existing approaches widely use Manhattan distance for pipe routing, but most SPI2 problems are not confined to regular grids. Therefore, improved SPI2 system performance requires more flexible representations that support deformable models in Euclidean spaces. In addition, optimal trajectory planning of pipe routing has not yet been developed in the existing interconnect routing research. Such a capability, and particularly one that operates in a multiphysics environment, would advance SPI2 design in several different ways. For example, it would allow the optimal path planning of a pipe that must pass through prescribed regions of space and physical fields while satisfying the connectivity constraints of the system.

(6) *Need for SPI2 data manipulation and interactive visualization tools*: 3D SPI2 design problems have heterogeneous elements, which makes the conceptualization of the design space quite challenging. Hence, there is a need for an interactive data manipulation tool aided by 3D visualization that helps practicing engineers follow and study intermediate stages of the optimization process, as well as compare multiple distinct SPI2 design solutions visually. Creating new tools of this sort will require answering questions such as how can one model and merge geometric, topological, and physical-design aspects along with constraint spaces. These challenges may require investigation into new meshing techniques, CAD representations, human data manipulation tools, and other candidate SPI2 design support elements.

(7) *Lack of proper integration of human factors into SPI2 engineering design*: For holistic SPI2 design, human-informed design is required for developing higher quality solutions to provide a competitive advantage, improve end-user

experience, and increase productivity and operational efficiency of product design pipelines. For greater effectiveness and smoother industry adoption, new SPI2 design research methods and tools must consider human factors such as physical limitations of designers (size or abilities), cognitive nature and capacities, human safety and risk prevention, consumer needs, training, and other related elements.

4.1 Associated Challenges. There are several challenges related to SPI2 research that should be addressed to support the holistic design methods envisioned in this article:

(1) Both 3D component placement and interconnect routing are NP-Hard problems. Therefore, as the scale and complexity of the system increase, the number of possible solutions explodes combinatorially, increasing decision-making costs significantly. The 3D topological space is vast and challenging to navigate as there can be infinite design options depending on the tuning parameters. Therefore, it is essential to have sampling strategies that can cover the design space thoroughly and efficiently.

(2) The 3D-SPI2 problem is a highly nonlinear optimization problem that simultaneously addresses component placement, routing, and physics performance evaluation. Therefore, there is a greater possibility of encountering local solutions with continuous spatial or parameter tuning when compared to design optimization of individual SPI2 problem elements.

(3) One key challenge in using gradient-based solution methods, such as the geometric projection method [152,163], is that changes in interconnect spatial topology may impact the lumped-parameter system models (such as fluid loops) in ways that either prevent stimulation of certain designs or at a minimum introduce non-smoothness.

(4) Creating design representations that can support topology, geometry, and physical aspects of the 3D SPI2 problem in a unified way is one of the most challenging aspects. Conventional methods address at most a pair of them while solving multiphysics optimization problems. Previous work exists where all three aspects are included but they are specific to their applications and do not generalize.

(5) SPI2 design automation tools should also consider the human perspective in all steps of the problem-solving process. In particular, industry practitioners who have vast experience in handling these complex systems possess valuable design knowledge that can be leveraged while developing SPI2 design automation frameworks. Incorporating human expertise into SPI2 automation methods, however, may introduce human biases or errors.

5 Discussion

3D component placement and 3D routing problems are individually NP-Hard problems, and solving the combined problem with multi-physics interactions and couplings between system elements is thus especially challenging. Ideally, the component placement and routing problems should be solved simultaneously to achieve system-optimal designs or at the very least iteratively so that both can be considered. However, a sequential effort, such as pack-then-route or vice versa, may not be capable of exploiting fully the design coupling between all sets of decisions, leading to suboptimality. The challenges are growing in significance as system compactness and performance requirements for engineering systems intensify. For example, commercial aircraft engines a few decades ago were larger compared to current designs; modern aircraft engine cores have a much smaller diameter and thus surface area, but must incorporate essentially the same externals, such as wires, pipes, and components, as older designs.

5.1 Spatial Packaging of Interconnected Systems Vision. Developing a holistic treatment of the SPI2 design problems will require answering several questions, including the following:

(1) How to characterize the SPI2 design space: What are its regions of interest? How do the feasible and infeasible design space regions compare with each other? What is the computational cost of determining the boundaries of the design space? What optimization methods are required to navigate the design space and how does one keep track of the explored regions?

(2) What design optimization frameworks are required to integrate different SPI2-related research areas together and search the SPI2 design space effectively, aiding both discrete and continuous decision-making?

(3) Does the SPI2 design space need to be specialized according to the properties of the SPI2 system, such as size, number and type of components and interconnects, physics, and product life-cycle cost value metrics, or can an inclusive design space be developed?

(4) How does the difficulty of the SPI2 design problem scale with increases in the number of components, constraints, or other complexity dimensions?

(5) What kinds of design tool functionality and readiness levels are required by the industry to adopt SPI2 design automation methods? What other factors influence the adoption of more capable SPI2 design methods and tools?

(6) What unified design parameterizations/representations are needed to solve the SPI2 optimization problems efficiently?

(7) How might various system-life-cycle value metrics such as manufacturing, maintenance, upgrade, overhaul, repair, and accessibility costs be incorporated as part of the SPI2 problem formulation and automated solution?

6 A Two-Stage Spatial Packaging of Interconnected Systems Design Automation Framework

Rather than making incremental progress on established methods for optimal spatial packaging and routing (PR), we previously developed a novel two-stage design framework [159], as shown in Fig. 6, for solving holistic SPI2 design problems. This framework can be viewed as a nested design optimization strategy [181,182] where the outer loop (Stage 1) navigates 3D spatial topological decisions and an inner loop (Stage 2) identifies optimal physics-based system performance for each unique spatial topology considered. The inner loop ensures a fair comparison of candidate topologies. Please note that the interconnect topologies remain fixed during this process. When efficient enumeration methods are used for the outer loop, the framework transforms into a sequential process. In Stage 1, unique interference-free spatial topologies are enumerated in a way that ensures component and interconnects non-intersection [183,184]. This non-intersection requirement often must be met for system model elements, such as lumped-parameter fluid circuit models, to be used without simulation failure. A variety of Stage 1 solution methods have been devised and tested, including shortest-path, force-directed layout, and spatial topology enumeration algorithms. These have the potential advantage of supporting trajectory-based 3D layout classification (i.e., whether interconnects move around objects clockwise/anticlockwise, go inside grooves, along a surface, attach at an angle, or go through hollow objects with holes (genus ≥ 1)).

The spatial topology design decision representation that is at the center of this successful SPI2 design method [168] utilizes spatial graphs, which are graphs embedded in metric spaces [166]. This fundamental advancement in formal SPI2 mathematical representations leverages existing spatial graph theory and computational machinery and furnishes the missing link needed to efficiently explore, enumerate, and identify unique 3D spatial topologies of SPI2 engineering systems. Other potentially useful mathematical representations likely exist, but this initial successful representation

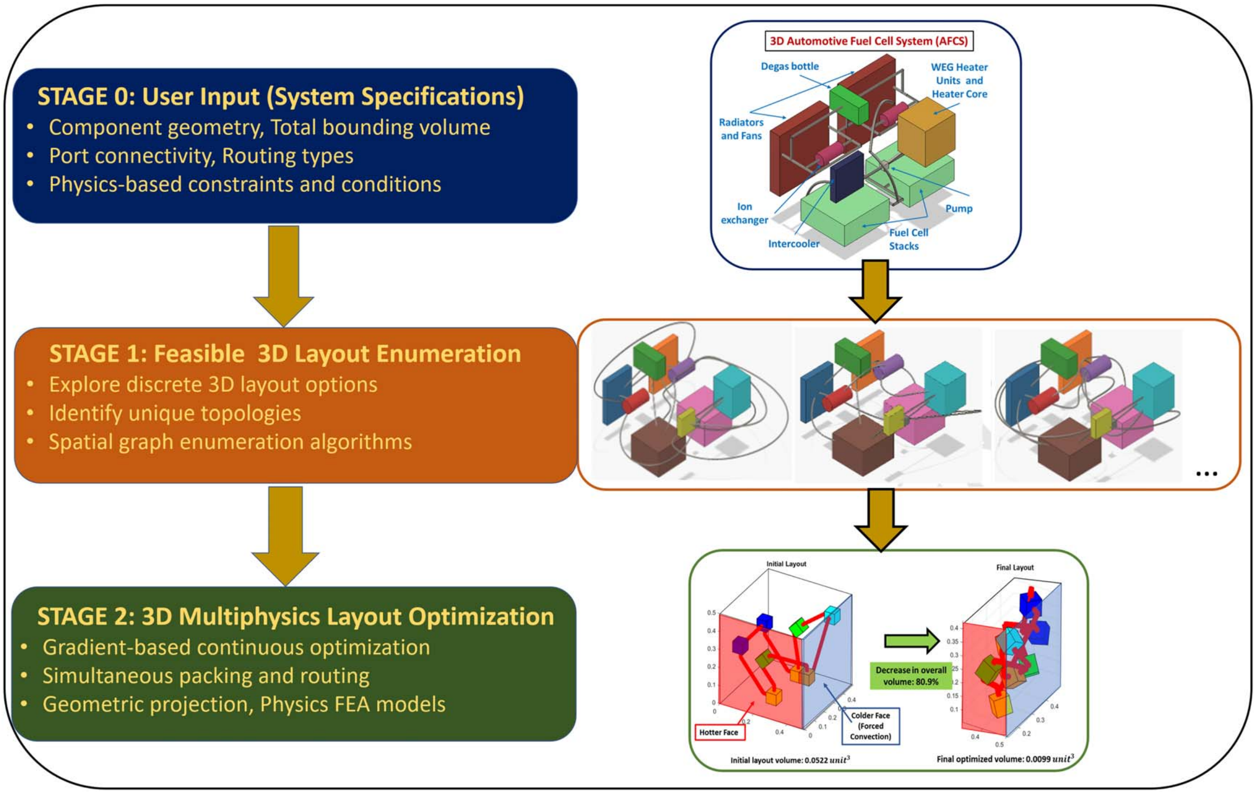

Fig. 6 Flowchart depicting the sequential two-stage SPI2 design automation framework. Stage 0 accepts user inputs and system specifications. Stage 1 uses the given system information to enumerate, identify, and generate unique discrete 3D spatial topologies (or configurations) of the engineering system. Stage 2 performs continuous multiphysics optimization for each unique spatial topology obtained from Stage 1. The final solutions of all the designs are then compared against each other according to a given performance metric to choose the best candidate. In this figure, we only show the initial and final layout of one such design candidate. Some of the optimization design variables in Stage 2 include locations and orientations of the components, lengths, diameters, bend radii, trajectories of interconnect routing, and other physical-design aspects of these elements.

is an important step toward holistic SPI2 design. Figure 3(*b*) shows how the spatial topology of a 3D SPI2 system can be captured by a spatial graph where components are the nodes, interconnections are the edges, and the ports are node valencies. In this two-stage method, all the combinatorial spatial graph descriptions up to some desired topological complexity level (e.g., crossing number) are enumerated for a given 3D system. A corresponding polynomial invariant, the Yamada polynomial, can then be calculated for all spatial graphs obtained from combinatorial permutations. The Yamada polynomials then are used as a tool to identify any duplicate spatial graph topologies, similar to isomorphisms for standard graphs. If two Yamada polynomials are equivalent, their corresponding graphs are topologically equivalent. A distilled smaller set of unique spatial embeddings is then used to automatically generate 3D geometric system models for Stage 2 parametric optimization.

Stage 2 begins with a spatially feasible 3D layout (obtained from Stage 1) and optimizes physics-based system performance with respect to component locations, interconnect paths, and other continuous design variables [159,160,163]. The continuous design variable representation enables the use of gradient-based methods to efficiently search the design space. Earlier, for 2D SPI2 problems in Ref. [159], we utilized a bar-based design representation with a differentiable geometric projection method (GPM). Bars have favorable geometric properties that can be exploited to represent both components and interconnects and solve the component placement and routing problems simultaneously for a fixed spatial topology. Differentiable GPMs have been demonstrated previously for several different physical domains, including structural optimization based on finite element analysis [152]. Barrier functions can be applied to prevent component/interconnect interference implicitly. We have also implemented 3D GPMs [164], where plates are used instead of bars as geometric primitives for components, obstacles, and interconnects. Optimization was performed with respect to plate location, shape, and orientation parameters, as opposed to discretized design representations (e.g., element-wise densities). This approach also has the benefit of simplifying the treatment of geometric constraints, physics-based and spatial constraints, and has been applied to several 3D test case problems. This method has been demonstrated for 2D and 3D test cases in Refs. [160,164,183]. To validate our SPI2 design automation method, understand its existing practical design limitations, and develop a more robust SPI2 automation method, the authors employed this two-stage method to solve a real-world SPI2 problem for an industrial application. In collaboration with practicing SPI2 design engineers, the two-stage method was further refined and extended to optimize the SPI2 design of an automotive fuel cell system (AFCS), the details of which are presented in Ref. [162]. Other valuable design representation and optimization strategies likely exist and will be the topic of future studies, but this new two-stage method is the first viable holistic SPI2 design automation method and establishes the exciting potential for a new era in SPI2 design research and practice.

The complete multiphysics optimization problem formulation for Stage 2 is given by

$$\min_{\boldsymbol{x}} \quad f(\boldsymbol{x},\ \boldsymbol{T}) \tag{1a}$$

$$\text{s.t.:} \quad \boldsymbol{g}_{\text{phys}}(\boldsymbol{x},\ \boldsymbol{T}) \le \boldsymbol{0} \tag{1b}$$

$$g_{\mathrm{dd}}(\boldsymbol{x}) \le \mathbf{0} \tag{1c}$$

$$g_{\mathrm{sd}}(\boldsymbol{x}) \le \mathbf{0} \tag{1d}$$

$$g_{\mathrm{ss}}(\boldsymbol{x}) \le \mathbf{0} \tag{1e}$$

$$\text{where:} \quad \boldsymbol{K}(\boldsymbol{x})\boldsymbol{T} = \boldsymbol{P}(\boldsymbol{x}) \tag{1f}$$

Here, $f(\boldsymbol{x}, \boldsymbol{T})$ is the objective function and $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{T})$ are constraint functions. In general, these functions may depend on both design ($\boldsymbol{x}$) and state ($\boldsymbol{T}$) variables. The function $f()$ can be any appropriate objective function, such as bounding box volume, overall system temperature distribution, energy efficiency, and holistic system performance measures. The constraints $\boldsymbol{g}_{\mathrm{phys}}(\boldsymbol{x}, \boldsymbol{T})$ are functions that depend both on design $\boldsymbol{x}$ and on state $\boldsymbol{T}$ variables, the latter of which requires a solution of the associated physics models. These physics-dependent constraints can involve limits on physical quantities, such as pressure head loss, device temperatures, fluid flowrates, pipe bend radii, material degradation metrics, stress, strain, and other properties. The interference constraints $\boldsymbol{g}_{\mathrm{dd}}(\boldsymbol{x})$, $\boldsymbol{g}_{\mathrm{sd}}(\boldsymbol{x})$, and $\boldsymbol{g}_{\mathrm{ss}}(\boldsymbol{x})$ prevent interference between two devices, one routing segment and one device and two routing segments, respectively. These constraints are independent of any physics models; they are all either explicit functions of the design variables or geometric interference calculations that depend on design. A combination of lumped-parameter and the finite element (distributed parameter) models was used to support efficient simulation of the multicomponent interconnect system. Sensitivity calculations for the objective and constraint functions are described in detail (for specific SPI2 problems) in Refs. [163,164].

One key challenge in using the gradient-based solution methods in this two-stage framework is that changes in interconnect spatial topology may impact the lumped-parameter system models (such as fluid loops) in ways that either prevent the simulation of certain designs or at a minimum introduce non-smoothness. We have identified multiple promising strategies for managing to interconnect topology decisions and plan to explore these options in conjunction with the continuous aspects of the problem. One strategy will be to utilize efficient graph-based enumeration strategies [179,185,186] to enumerate unique and feasible interconnect topology options and, then for each option, solve the continuous optimization problem. This has potential for scaling to large systems using machine learning trained on enumeration data. The second strategy is to investigate possible topology optimization techniques to allow interconnects to pass through each other while preserving model smoothness while preventing flow system simulation failure. In this way, we can relax the discrete topology design problem and absorb this task into our broader gradient-based design framework. Steps also need to be taken to assess and mitigate additional challenges such as local minima, which can arise when implementing relaxation methods.

6.1 Challenges Addressed and Limitations of the Two-Stage Approach. In summary, the two-stage SPI2 design automation approach is a new method that is still under development. It is one viable and flexible design approach that addresses some of the following challenges for holistic SPI2 design:(1) supports systematic exploration and efficient navigation through unique 3D discrete spatial topology options enabled through the use of spatial graph enumeration techniques; (2) the discrete and continuous aspects of the problem have been addressed in two-separated stages that are combined together in a sequence through a nested optimization approach; (3) the gradient-based geometric projection topology optimization method supports the simultaneous component placement, routing, and physics evaluation needed for Stage 2; (4) spatial graphs are utilized as flexible mathematical design representations for performing optimization; and (5) the method is scalable to larger systems through parallel processing and high-performance computing.

Although this method addresses several challenges of a holistic SPI2 design problem, some limitations to be overcome in the future include (1) complicated component geometry is not included in this framework. Investigation of candidate unified geometric representations is being performed to enable the treatment of arbitrary geometric shapes while supporting physics-based analysis and interference detection. (2) It is very difficult to ascertain global optimal solutions as the objective and constraint functions often are very nonlinear, the holistic SPI2 problem involves mixed (discrete and continuous) variables, and the design space can be vast as system complexity increases. The current optimization algorithm performs an exhaustive search but must be capable of reliably finding global optima for Stage 2 to ensure a fair comparison of spatial topology candidates. (3) Life-cycle costs and operational management objectives and constraints are not yet included in automated SPI2 design methods. These features are being incorporated into the current two-stage framework as part of ongoing work. (4) A minor limitation in using a gradient-based optimization strategy is that changes in interconnect topology can impact the lumped-parameter system models (such as fluid loops) in ways that either prevent simulation of certain designs or at a minimum introduce non-smoothness. One possible remedy is to create techniques that allow interconnects and components to pass smoothly through each other at intermediate Stage 2 optimization iterations without fluid mixing.

In summary, the two-stage design framework is a sequential two-step process, and the two stages (spatial topology enumeration and continuous multiphysics optimization) are separate and not fully integrated. A method that can simultaneously handle discrete topology changes (Stage 1), and continuous changes in geometry and physics during optimization (Stage 2) are still under investigation. There are three different aspects addressed here—spatial topology, geometry, and physics. Stage 1 deals with the discrete spatial topology decisions only, such as how interconnects pass across each other and the solid components. Whereas Stage 2 of this framework is a completely integrated method within itself, geometry and physics coupling is considered together through a continuous gradient-based optimization process. In Stage 2, any change in geometry (location or orientation of component, and diameter, length, and trajectory of interconnects) directly affects physics and vice versa during every evaluation. For example, as illustrated in Refs. [163,164], if the pressure drop in the network is becoming high and may violate the total head loss constraint, the pipes become longer and smoother at the bends, or if the temperature of the system elements may exceed limits, then the components and pipes move toward the relatively colder region in the domain and orient themselves to get effectively cooled.

7 Conclusion

Effective design automation strategies are key to meeting the demands of present and future needs for 3D physics-based system packaging or SPI2 problems. Systematic, flexible, and efficient design methods with the ability to explore and access new configurations are essential for achieving better system performance, compactness, and life-cycle cost across different engineering industries. Effective methods will support adjustments that can be made easily as the system requirements change over time. An important potential benefit of realizing such methods is the reduction in design time and resources required to solve 3D component placement and routing problems, enabling greater tailoring of designs to enhance performance for unique applications, while reducing design effort.

Creating a body of knowledge within the 3D spatial packaging of interconnected systems' space is central to solving important problems throughout the engineering product life cycle, from manufacturing, to assembly, maintenance, diagnosis, repair, and retrofit. Simple designs are typically employed in current practice to keep these problems tractable; providing a means to reason in this complex space offers an unprecedented opportunity to increase product performance and packaging density, while leveraging advanced manufacturing methods and automated assembly methods. This article reviews the technical groundwork, defines the shape and bounds of this knowledge domain, and specifies an initial set of key areas to jumpstart the engineering research community's efforts in this field. Some of the existing critical gaps that prevent the creation and successful application of design automation methods to industry-relevant holistic SPI2 problems are outlined, and associated challenges are addressed. Finally, some larger SPI2 design research questions are presented and one viable SPI2 design automation approach (two-stage design framework) developed by the authors is discussed as an example of handling SPI2 research problems. This article is an initial introduction to the SPI2 class of problems to promote discussion, help realize the societal impact, and catalyze a surge of research activity in this domain. In the future, these topics will be demonstrated in more depth with illustrative examples, design representations, and new SPI2 solution methods.

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Conflict of Interest

There are no conflicts of interest.

Data Availability Statement

No data, models, or code were generated or used for this paper.

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